

Clearing an Orthogonal Polygon Using Sliding Robots

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Abstract

In a multi-robot system, a number of autonomous robots would sense, communicate, and decide to move within a given domain to achieve a common goal. In this paper, we consider a new variant of the pursuit-evasion problem in which the robots (pursuers) each move back and forth along an orthogonal line segment inside a simple orthogonal polygon P . A point p can be covered by a sliding robot that moves along a line segment s , if there exists a point $q \in s$ such that \overline{pq} is a line segment perpendicular to s . In the pursuit-evasion problem, a polygonal region is given and a robot called a pursuer tries to find some mobile targets called evaders. The goal of this problem is to design a motion strategy for the pursuer such that it can detect all the evaders. We assume that P includes unpredictable, moving evaders that have unbounded speed. We propose a motion-planning algorithm for a group of sliding robots, assuming that they move along the pre-located line segments with a constant speed to detect all the evaders with unbounded speed.

1998 ACM Subject Classification I.3.5 Computational Geometry and Object Modeling

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1 Introduction

The mathematical study of the “pursuit-evasion” problem was first considered by Parson [11]. After that, the watchman route problem was introduced as a variation of the art gallery problem, which consists of finding static evaders in a polygon. The visibility-based motion-planning problem was introduced in 1997 by Lavalley et al. [6]. The aim was to coordinate the motions of one or more robots (pursuers) that have omnidirectional vision sensors to enable them to eventually “see” an evader that is unpredictable, has an unknown initial position, and is capable of moving arbitrarily fast. The process of detecting all evaders

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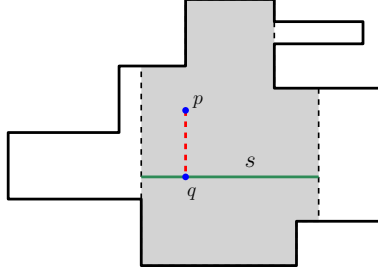


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is also known as clearing the polygon. The pursuit-evasion problem has a broad range of applications such as air traffic control, military strategy, and trajectory tracking [6].

In 2011, Katz and Morgenstern introduced sliding camera guards for guarding orthogonal polygons [5]. We define the “sliding robots” to be the same as the sliding cameras, where the robot r_i would travel back and forth along an axis-aligned segment s inside an orthogonal polygon P . A point p is seen by s_i if there exists a point $q \in s_i$ such that \overline{pq} is a line segment perpendicular to s_i and is completely inside P . The set of all points of P that can be seen by s_i is its sliding visibility polygon (see Fig.1). The point p is seen by r_i if $r_i = q$.



■ **Figure 1** The shaded area shows the sliding visibility polygon of s .

According to the visibility-based motion-planning problem and the sliding robots, we study the new version of planning the motions for a group of robots for clearing an orthogonal polygon when robots are modeled as sliding cameras. The given orthogonal polygon P has unpredictable, moving evaders with unbounded speed. Motion planning for a group of sliding robots to clear P means presenting the sequence of motions for the sliding robots such that any evader is viewed by at least one robot. Moreover, a set of line segments, S , is given such that the union of their sliding visibility polygons is P .

Previous Works

Generally, in the pursuit-evasion problem, the pursuer is considered as an l -searcher with l flashlights and rotates them continuously with a bounded angular rotation speed [13]. Thus, an ∞ -searcher (also known as an omnidirectional searcher) is a mobile robot equipped with a 360° view sensor for detecting evaders. Lavalley et al. proposed the first algorithm for solving the pursuit-evasion problem for an l -searcher [6]. They decomposed P into cells based on visibility properties and converted the problem to a search on an exponential-sized information graph. Durham et al. [2] addressed the problem of coordinating a team of mobile robots with limited sensing and communication capabilities to detect any evaders in an unknown and multiply connected planar environment. They proposed an algorithm that guarantees the detection of evaders by maintaining a complete coverage of the frontier between cleared and contaminated regions while expanding the cleared region.

The art gallery problem is a classical and old problem in computational geometry. Over the years, many variants of this problem have been studied [10, 14, 4, 12]. Most of these have been proved to be NP-hard [7], containing the problem when the target region is a simple orthogonal polygon, and the goal is to find the minimum number of vertex guards to guard the entire polygon (e.g., [10, 12]). Some types of them, which consider the limited model of visibility, use polynomial time algorithms [9, 15].

The study of the art gallery problem based on the sliding camera was started in 2011 by Katz and Morgenstern [5]. They studied the problem of guarding a simple orthogonal

polygon using minimum-cardinality sliding cameras (MCSC). They showed that, when the cameras are constrained to travel only vertically inside the polygon, the MCSC problem can be solved in polynomial time. They also presented a two-approximation algorithm for this problem when the trajectories that the cameras travel can be vertical or horizontal and the target region is an x -monotone orthogonal polygon. They left the computation of the complexity of the MCSC problem as an open problem. In 2013, Durocher and Mehrabi [3] studied these two problems: the MCSC problem and the minimum-length sliding camera (MLSC) problem, where the goal was to minimize the total length of the trajectories along which the cameras travel. They proved that the MCSC problem is NP-hard, where the orthogonal polygon can have holes. They also proved that the MLSC problem is solvable in polynomial time even for orthogonal polygons with holes. In 2014, Durocher *et al.* [8] presented an $\mathcal{O}(n^{2.5})$ -time $(7/2)$ -approximation algorithm for solving the MCSC problem in simple orthogonal polygons. In 2014, De Berg *et al.* [1] presented a linear-time algorithm for solving the MCSC problem in an x -monotone orthogonal polygon. The complexity of this problem remains as an open problem.

Our Result

Our aim is to plan the motions for a group of robots that move along the line segments of S and find all unpredictable evaders such that the number of robots used is the cardinality of S . Owing to the difficulty of having multiple cooperating robots executing common tasks, we store some information (e.g., the status of some nearby regions that shows whether the regions have been cleared by some robots) on each reflex vertex.

We assume that the robots have the map of the environment and that they are capable of broadcasting a message (e.g., a region that is supposed to get cleared) to all the other robots by sending signals. This way, the robots can have some communications with each other to maintain the coordination process.

The best result of our algorithm is that, if S is a set of MCSCs that guard the whole P , then our algorithm will detect all evaders with the ***minimum number of sliding robots***.

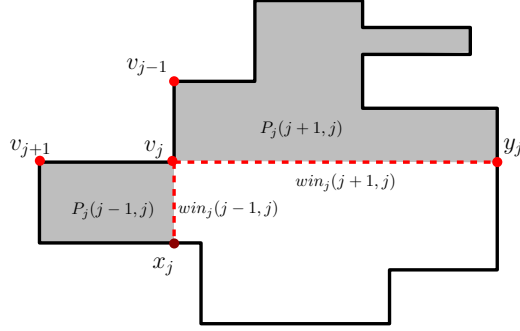
2 Preliminaries and Notations

Let P be an orthogonal polygon and $V(P) = \{v_1, v_2, \dots, v_n\}$ be the set of all vertices of P in counterclockwise order. We consider $V_{ref}(P)$ to be all of the reflex vertices of P and assume a general position such that no four reflex vertices are collinear. Suppose that P_1 is a sub-polygon of P whose boundary is from a to b (a and b are points on the boundary of P) in counterclockwise order. Then, we show P_1 by (a, b) .

Let v_j be a reflex vertex of P . v_j has two edges, $e_{j-1} = \overline{v_{j-1}, v_j}$ and $e_j = \overline{v_j, v_{j+1}}$, that can be extended inwardly until they reach the boundary of P . We call these extensions as the windows of v_j and show them as $win_j(j-1, j) = \overline{v_j x_j}$ and $win_j(j+1, j) = \overline{v_j y_j}$, respectively (x_j and y_j are two points on the boundary). $win_j(j-1, j) = \overline{v_j x_j}$ and $win_j(j+1, j) = \overline{v_j y_j}$ are two line segments whose endpoints are on the boundary of P . $win_j(j-1, j)$ partitions P into two sub-polygons. Let $P_j(j-1, j)$ be a sub-polygon that consists of v_{j+1} , and let $P'_j(j-1, j)$ be $P \setminus P_j(j-1, j)$. Therefore, $P_j(j-1, j)$ and $P'_j(j-1, j)$ are (v_j, x_j) and (x_j, v_j) , respectively.

Similarly, let $P_j(j+1, j)$ be a sub-polygon that is separated from P by $win_j(j+1, j)$ and consists of v_{j-1} , and let $P'_j(j+1, j)$ be a sub-polygon that includes v_{j+1} . Therefore, $P_j(j+1, j)$ and $P'_j(j+1, j)$ are (y_j, v_j) and (v_j, y_j) , respectively. Let L be the set of all

windows of P . L partitions P into orthogonal rectangles.



■ **Figure 2** Shown are the windows and the sub-polygons of v_j .

3 The Proposed Algorithm

In this section, we present an algorithm for solving the pursuit-evasion problem using sliding robots. Assume that an orthogonal polygon P and a set of orthogonal line segments $S = \{s_1, s_2, \dots, s_k\}$ are given. We present a path-planning algorithm for finding the unpredictable evaders using a set of sliding robots $R = \{r_1, r_2, \dots, r_k\}$ in which r_i can move along the line segment s_i .

To distribute the movements of the robots, we define the “event points” as below:

► **Definition 1.** An **event point** happens when r_i sees a reflex vertex, sees a waiting sliding robot, or reaches an endpoint of s_i .

Overview of the Algorithm

Our algorithm has five steps. The “start step,” the “decision step,” the “sending a signal,” the “move back-and-update step,” and the “termination step.”

To present our path-planning method, we start with an arbitrary sliding robot $r_i \in R$, which is on $s_i \in S$ (start step). r_i starts moving from one endpoint of s_i . When r_i reaches an event point, it updates the cleared sub-polygons. By the time that $D_i(2)$ becomes empty and $D_i(1) \neq \emptyset$, r_i moves back along s_i (move back-and-update step). Moreover, at each event point, r_i stops and, according to the cleared sub-polygons of P , decides to continue its movement or send a signal to the other robots to clear a specific sub-polygon of P (decision step). When r_i sends a signal to the other robots to clear a sub-polygon P_1 , a robot that can clear some parts of P_1 starts moving along its corresponding line segment (sending a signal step). When all parts of P become cleared, the algorithm is finished (termination step).

Details of the Algorithm

Now, we explain the steps of the algorithm in detail. We store the status of the regions in their corresponding reflex vertices, which are updated by the robots during the movements to keep track of the contaminated regions, which is helpful in the decision-making process.

For each $v_j \in V_{ref}(P)$, we store an array called $FF_j(i)$ ($1 \leq i \leq 4$) of size four in which the cells (of type Boolean) indicate whether the sub-polygons $P_j(j-1, j)$, $P_j(j+1, j)$,

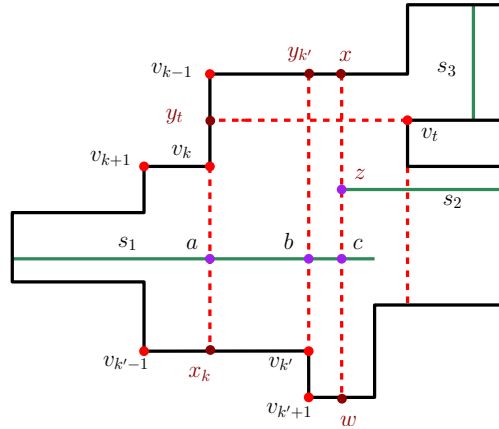
$P'_j(j-1, j)$, and $P'_j(j+1, j)$ are cleared (true), respectively. Initially, we assume that all parts of P are contaminated; therefore, $\forall 1 \leq i \leq 4, FF_j(i) = false$.

For each $r_i \in R$, we consider a triple storage, which is called $D_i(j) 1 \leq j \leq 3$. Each storage includes an interval such as (a, b) , which indicates the boundary of P between a and b in counterclockwise order. The first storage, $D_i(1)$, indicates the cleared sub-polygon of P by r_i (partly or completely). The second storage, $D_i(2)$, indicates the sub-polygon of P that should be cleared by r_i (partly or completely). The third storage, $D_i(3)$, specifies the sub-polygon that should be cleared until r_i continues its movement. In the case where r_i is waiting, $D_i(3)$ is not empty. Initially, for each $r_i \in R$, $D_i(1) = D_i(2) = D_i(3) = \emptyset$.

Start Step

As mentioned earlier, we start with one of the endpoints of an arbitrary s_i (r_i can move along s_i).

- If r_i starts from an endpoint that is on the boundary, r_i can see two consecutive vertices (suppose the endpoint is on the edge $e_k = v_k v_{k+1}$).
 1. If v_k and v_{k+1} are convex, then r_i starts clearing P by its movement and updates $D_i(1) = (v_k, v_{k+1})$. r_i continues its movement along s_i until an event point happens. At these times, r_i stops, updates $D_i(1)$ and $D_i(2)$, and makes a decision for its movement (decision step).
 2. If at least one of v_k or v_{k+1} is a reflex vertex, then r_i cannot start clearing P ; it therefore stops and waits on the endpoint to make a decision (decision step).
- If r_i wants to start from an endpoint that is not on the boundary, then r_i cannot start clearing P ; it therefore stops and waits on the endpoint (decision step). Suppose that the maximal normal line segment to s_i that passes through r_i is lr . Let x and w be the first intersection of lr at the boundary of two sides. s_i can be inside the sub-polygon corresponding to (x, w) or (w, x) . Assume that s_i is inside (w, x) . Therefore, r_i sends a signal to the other robots to clear (x, w) , and $D_i(3) = (x, w)$ (sending a signal step). As shown in Fig.3, if r_i wants to start from z , it stops and sends a signal to the other robots to clear the sub-polygon corresponding to (x, w) .



■ **Figure 3** r_1, r_2 , and r_3 moving along s_1, s_2 , and s_3 , respectively.

Move Back-and-Update Step

Assume that r_i moves along s_i . When an event point happens, r_i updates $D_i(1)$ (increases the cleared region) and $D_i(2)$ (decreases the sub-polygon that should be cleared). At each

time that $D_i(2)$ becomes empty (and $D_i(1) \neq \emptyset$), r_i moves back along s_i . It moves back until it sees a waiting robot or reaches an endpoint of s_i . When r_i sees a reflex vertex v_k during its movement, it updates $FF_k(j)$ for $1 \leq j \leq 4$ as detailed below:

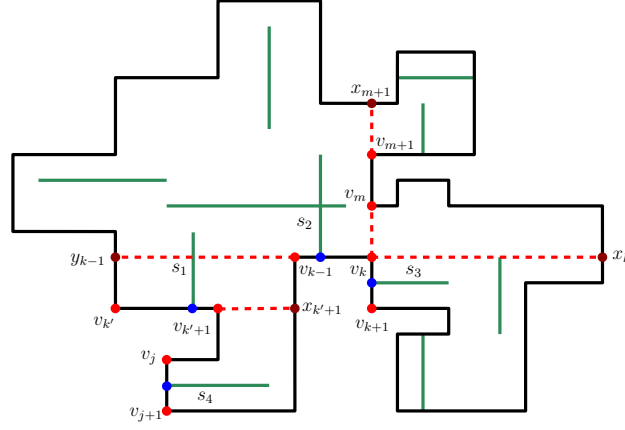
- If the endpoints of $win_k(k-1, k)$ and v_{k+1} are inside the sub-polygon indicated by $D_i(1)$ (if $D_i(1) = (v_k, x_k)$, then $v_{k+1} \in D_i(1)$), then the sub-polygon $P_k(k-1, k)$ is cleared and r_i updates $FF_k(1) = true$ (see Fig.3, when r_1 moves back from left to right and reaches a).
- If $D_i(1) = (x_k, v_k)$, then r_i updates $FF_k(3) = true$ (see Fig.3, when r_1 moves back from right to left and reaches a).
- If $D_i(1) = (y_k, v_k)$, then r_i updates $FF_k(2) = true$ (see Fig.3, when r_1 moves back from left to right and reaches b).
- If $D_i(1) = (v_k, y_k)$, then r_i updates $FF_k(4) = true$ (see Fig.3, when r_1 moves back from right to left and reaches b).

As we explained earlier, r_i moves back until it finishes clearing ($D_i(2) = \emptyset$). While it is moving back, if r_i sees its corresponding waiting robot (supposedly r_j) and $D_i(1) = D_j(3)$, then $D_i(2) = \emptyset$. Therefore, r_i updates $D_j(3) = \emptyset$, $D_j(1) = D_j(1) \cup D_i(1)$, and $D_j(2) = D_j(2) \cup D_i(1)$. Since $D_i(2)$ is empty, r_i finishes its clearing and r_j starts moving back (see Fig.3; when r_1 moves back from left to right and reaches c , it updates the information of r_2 , and r_2 moves back). r_j can be collinear with the endpoint of s_i . Moreover, if r_i sees any reflex vertex v_k , r_i updates $FF_k(j)$ for $1 \leq j \leq 4$ as explained above and continues moving back.

Decision Step

When r_i stops and waits, it makes a decision and performs the following:

1. If r_i is on the endpoint of s_i (let ep be the endpoint), then
 - a. If ep is on the boundary of P (on the edge $(e_k = \overline{v_k v_{k+1}})$), then
 - If $v_k \in V_{ref}(P)$ and $P_k(k+1, k)$ is contaminated ($FF_k(2) = false$), then $P_k(k+1, k)$ should be cleared. Therefore, r_i sends a signal to the other robots to clear $P_k(k+1, k)$ and updates $D_i(3) = (y_k, v_k)$ (As mentioned in Section 2, y_k and v_k are two endpoints of $win_k(k+1, k)$, and since $P_k(k+1, k)$ includes v_{k-1} , $D_i(3)$ is from y_k until v_k in counterclockwise order).
For an example, see Fig.4; assume that r_3 or r_2 is on the blue point of s_3 and s_2 , respectively.
 - Else if $P_k(k+1, k)$ is cleared ($FF_k(2) = true$), then $D_i(1) = D_i(1) \cup (y_k, v_k)$ and $D_i(2) = D_i(2) \setminus (y_k, v_k)$.
 - If $v_{k+1} \in V_{ref}(P)$ and $P_{k+1}(k, k+1)$ is contaminated ($FF_{k+1}(1) = false$), then $P_{k+1}(k, k+1)$ should be cleared. Therefore, r_i sends a signal to the other robots to clear $P_{k+1}(k, k+1)$ and updates $D_i(3) = (v_{k+1}, x_{k+1})$.
For an example, see Fig.4; assume that r_1 or r_2 is on the blue point of s_1 and s_2 , respectively.
 - Else if $P_{k+1}(k, k+1)$ is cleared ($FF_{k+1}(1) = true$), then $D_i(1) = D_i(1) \cup (v_{k+1}, x_{k+1})$ and $D_i(2) = D_i(2) \setminus (v_{k+1}, x_{k+1})$.
 - If at least one of v_k and v_{k+1} is a reflex vertex, then ep is on $l(j) \in L$. If $l(j)$ includes two consecutive reflex vertices v_m, v_{m+1} , where $m \neq k$ (suppose that the nearest one to r_i is v_m), then
For an example, See Fig.4; assume that r_3 is on the blue point of s_3 .
 - i. If $P_{m+1}(m, m+1)$ is contaminated ($FF_{m+1}(1) = false$), then r_i sends a signal to the other robots to clear $P_{m+1}(m, m+1)$ and updates $D_i(3) = (v_{m+1}, x_{m+1})$



■ **Figure 4** r_1, r_3 , and r_4 moving along s_1, s_3 , and s_4 , respectively

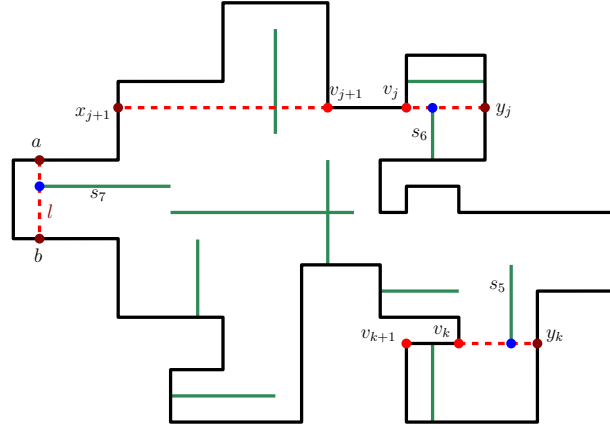
- ii. Else ($FF_{m+1}(1) = true$), $D_i(1) = D_i(1) \cup (v_{m+1}, x_{m+1})$ and $D_i(2) = D_i(2) \setminus (v_{m+1}, x_{m+1})$.
- If v_k and v_{k+1} are convex, then
 - i. If r_i wants to start moving from ep (if $D_i(1) = \emptyset$), then r_i updates $D_i(1) = (v_k, v_{k+1})$ and starts moving along s_i .
 - ii. If r_i reaches the endpoint of s_i (if $D_i(1) \neq \emptyset$), then $D_i(2)$ is \emptyset and r_i moves back.
- b. If ep is not on the boundary of P and ep is collinear by at least one reflex vertex, then ep is on $l(j) \in L$. Therefore,
 - If $l(j)$ consists of one reflex vertex v_k (assume that the consecutive vertex of v_k on $l(j)$ is v_{k+1}) and s_i is inside $P_k(k+1, k)$, then

For an example, see Fig.5; assume that r_5 is on the blue point of s_5 .

 - i. If $P'_k(k+1, k)$ is contaminated ($FF_k(4) = false$), then r_i sends a signal to the other robots to clear $P'_k(k+1, k)$ and updates $D_i(3) = (v_k, y_k)$.
 - ii. Else ($FF_k(4) = true$), $D_i(1) = D_i(1) \cup (v_k, y_k)$ and $D_i(2) = D_i(2) \setminus (v_k, y_k)$.
 - Else if s_i is inside $P'_k(k+1, k)$, then
 - i. If $P_k(k+1, k)$ is contaminated ($FF_k(2) = false$), then r_i sends a signal to the other robots to clear $P_k(k+1, k)$ and updates $D_i(3) = (y_k, v_k)$.
 - ii. Else ($FF_k(2) = true$), $D_i(1) = D_i(1) \cup (y_k, v_k)$ and $D_i(2) = D_i(2) \setminus (y_k, v_k)$.
 - If $l(j)$ consists of two consecutive reflex vertices v_k and v_{k+1} (suppose that the nearest one to ep is v_k , and s_i is inside $P_k(k+1, k)$, then

For an example, see Fig.5; assume that r_6 is on the blue point of s_6 .

 - i. If $P_{k+1}(k, k+1)$ is contaminated ($FF_{k+1}(1) = false$), then r_i sends a signal to the other robots to clear it ($P_{k+1}(k, k+1)$) and updates $D_i(3) = (v_{k+1}, x_{k+1})$.
 - ii. Else ($P_{k+1}(k, k+1)$ is cleared ($FF_{k+1}(1) = true$)) r_i sends a signal to the other robots to clear $P'_{k+1}(k, k+1) \cap P'_k(k+1, k)$ and updates $D_i(3) = (x_{k+1}, y_k)$.
 - Else if s_i is inside $P'_k(k+1, k)$, then
 - i. If $P_{k+1}(k, k+1)$ is contaminated ($FF_{k+1}(1) = false$), then r_i sends a signal to the other robots to clear it ($P_{k+1}(k, k+1)$) and updates $D_i(3) = (v_{k+1}, x_{k+1})$.
 - ii. If $P_k(k+1, k)$ is contaminated ($FF_k(2) = false$), then r_i sends a signal to the other robots to clear it and updates $D_i(3) = (y_k, v_k)$.
 - iii. If $P_{k+1}(k, k+1)$ and $P_k(k+1, k)$ are cleared ($FF_{k+1}(1) = true$ and $FF_k(2) = true$), then $D_i(1) = D_i(1) \cup (y_k, x_{k+1})$ and $D_i(2) = D_i(2) \setminus (y_k, x_{k+1})$.



■ **Figure 5** r_5, r_6 , and r_7 moving along s_5, s_6 , and s_7 , respectively

- c. If ep is not on the boundary of P and ep is not collinear by any reflex vertex, then suppose that the maximal orthogonal line segment normal to s_i at ep is l and let a and b be two endpoints of l . l partitions P into two sub-polygons. One of them consists of s_i . Therefore, r_i sends a signal to the other robots to clear the sub-polygon that does not include s_i and that is between a and b (r_i updates $D_i(3)$ depending on its position to $D_i(3) = (a, b)$ or $D_i(3) = (b, a)$).
For an example, see Fig.5; if r_7 is on the blue point of s_7 , then the sub-polygon that is between (a, b) in counterclockwise order should be cleared.
2. Else if r_i sees at least one reflex vertex (r_i is on $l(j) \in L$), then
 - a. If there are no two consecutive reflex vertices on $l(j)$, then r_i continues its movement along s_i .
 - b. If there are two consecutive reflex vertices v_k, v_{k+1} on $l(j)$ (suppose that the nearest one to r_i is v_k), then r_i decides as below:
For an example, see Fig.6; assume that r_1 is on the point p of s_1 .
 - If $P_{k+1}(k, k+1)$ is cleared ($FF_{k+1}(1) = \text{true}$), then r_i updates $D_i(1) = D_i(1) \cup (v_{k+1}, x_{k+1})$ and $D_i(2) = D_i(2) \setminus (v_{k+1}, x_{k+1})$, and then continues its movement along s_i .
 - If $P_{k+1}(k, k+1)$ is contaminated ($FF_{k+1}(1) = \text{false}$), then $P_{k+1}(k, k+1)$ should be cleared. Therefore, r_i waits and sends a signal to the other robots to clear $P_{k+1}(k, k+1)$ and updates $D_i(3) = (v_{k+1}, x_{k+1})$.

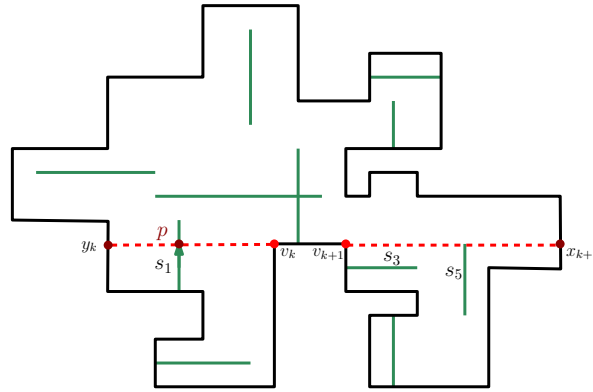
Waiting and Sending a Signal Step

Assume that r_i waits and sends a signal to the other robots to clear sub-polygon P_1 , which is between a and b in counterclockwise order ($D_i(3) = (a, b)$).

When r_i sends a signal, a robot that can clear some portions of P_1 consisting of a starts clearing. At each time, one robot is clearing. Suppose that r_j sees a and can start clearing P_1 . Therefore, r_j updates $D_j(2) = D_i(3)$.

If r_j is outside of P_1 , r_j starts clearing from a and $D_j(1)$ is the intersection of the boundary of P_1 and the orthogonal line segment that passes through a and intersects s_j . Therefore, r_j starts its movement (see Fig.6). Otherwise (r_j is inside P_1), r_j starts clearing from one of its endpoints (for an example, see Fig.5; if r_5 is on the blue point of s_5 , $P_{k+1}(k, k+1)$ should be cleared).

Suppose that v_k is a reflex vertex and that $FF_k(x) = \text{false}$ (let P_1 be the corresponding sub-polygon of $FF_k(x)$); suppose also that r_j is a robot that is waiting until P_1 becomes



■ **Figure 6** When r_1 reaches p and $FF_{k+1}(1) = false$, r_3 moves along s_3 from its left endpoint (or r_5 moves along s_5 from its upper endpoint).

cleared. At the time that r_i updates $FF_k(x)$ to *true*, r_i finishes its clearance and updates $D_j(1) = D_j(1) \cup D_i(1)$ and $D_j(2) = D_j(2) \setminus D_i(1)$. Then, r_j continues its movement.

Termination Step Algorithm

We assume that, initially, all parts of P are contaminated and $\forall_{r_i \in R} D_i(1) = \emptyset$. Because of our algorithm, a robot can move and clear some parts of P at any time. When there is no waiting robot ($\forall_{r_i \in R} D_i(3) = \emptyset$), all robots have cleared their corresponding sub-polygons ($\forall_{r_i \in R} D_i(2) = \emptyset$), and all parts of P have been cleared ($\bigcup_{i=1}^{|R|} D_i(1) = P$), the motion-planning algorithm is finished.

4 Analysis

In this section, we shall prove that the proposed algorithm is deadlock free. Since S guards all parts of P , then the algorithm will be terminated. Then, we will show that, starting with any arbitrary sliding robot, the algorithm can clear P completely.

► **Lemma 2.** *The proposed algorithm is deadlock free.*

Proof. Assume that r_i is waiting for sub-polygon P_i to be cleared by a sequence of robots. Inside P_i , r_j may be waiting for sub-polygon P_j to be cleared. Therefore, there may exist a chain of waiting robots, say, $r_{seq}(i) = \langle r_j, r_t, \dots, r_m \rangle$, for clearing P_i . If $r_i \in r_{seq}(i)$, a deadlock occurs and the algorithm will not get terminated. Therefore, we shall show that the relation $r_i \in r_{seq}(i)$ will never become valid.

Owing to the definition of the window and its corresponding sub-polygons, when r_i waits for the clearance of P_i , it cannot see any points of P_i , except its window. Since the sub-polygons corresponding to the other robots in $r_{seq}(i)$ are inside P_i , none of the waiting robots in $r_{seq}(i)$ can wait for r_i . Hence, the algorithm is deadlock free. ◀

► **Lemma 3.** *A simple orthogonal polygon can be completely cleared starting with an arbitrary sliding robot.*

Proof. Assume that we start with an arbitrary robot r_i . Because of Lemma 2, the proposed algorithm is deadlock free. Moreover, since S guards all parts of P , the termination step

will happen. Based on the termination step, the relation $\bigcup_{i=1}^{|S|} D_i(1) = P$ becomes valid; therefore, there is no contaminated point in P and the polygon gets cleared completely. ◀

► **Theorem 4.** *Let P be a simple orthogonal polygon consisting of unpredictable evaders, and let S be a set of line segments such that the union of their sliding visibility polygons is P . We can propose a motion-planning algorithm for a group of sliding robots that move along the line segments of S and find all evaders such that the number of sliding robots used is at most the cardinality of S .*

► **Corollary 5.** *If S is the set of minimum cardinality sliding cameras that guard the whole P , then our algorithm clears P with the minimum number of sliding robots.*

5 Conclusion

In this paper, we have proved that, in the case of having a known environment for sliding robots, there exists an algorithm for planning the motions of a group of sliding robots to detect all the unpredictable moving evaders that have unbounded speed. We assume that the speed of the sliding robots is unbounded ($\neq \infty$). We use a set of line segments S where the sliding robots move along. In the case where S is a set of minimum-cardinality sliding cameras that guard P , the proposed algorithm uses the minimum number of sliding robots to clear P .

Investigating the problem in which the environment is unknown to the robots, and in which the robots could only plan their motions based on the local visible area, would be challenging. Additionally, letting the robots send information only to those that are visible to them may make the problem more usable in real-life multi-robot systems.

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